

FIELD THEORY BASED S-PARAMETER ANALYSIS OF CIRCULAR RIDGED WAVEGUIDE DISCONTINUITIES

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Abstract - This paper describes a radial mode matching algorithm for the S-parameter computation of circular ridged waveguide (CRW) discontinuities. By shaping the ridge section conically, the coupling integrals in the discontinuity region can be solved analytically. This approach avoids the use of a mixed coordinate system and leads to a fast algorithm which is suitable for computer-aided design of CRW components like filters, septum polarizers and waveguide transformers. A comparison between measured and computed S-parameters shows excellent agreement. To illustrate the capability of the algorithm, a circular double ridged waveguide filter in a below cutoff waveguide and a CRW transformer have been optimized.

I Introduction

Application of circular ridged waveguides (CRW) can be found in many components like filters, matching networks, polarizers and circulators. In the design of these components it is important to characterize accurately the transition between the empty circular waveguide and the CRW section, as well as the interaction between subsequent discontinuities. For various types of CRW's an eigenvalue analysis has been performed in [5,6] using a radial mode matching technique. However, this is only the first step towards field theory based design of CRW components. To complete the analysis presented in [5,6], this paper introduces an S-parameter analysis of circular double ridge discontinuities in which the interaction between fundamental and higher order modes is included. Our objectives is a fast and accurate algorithm which can be implemented in an optimization routine for the design of CRW components. For this reason the ridges are shaped conically (Figure 1) to avoid a mixed coordinate system at the discontinuity between the empty and the ridged waveguide section. In this case the coupling between the fundamental TE and higher order TE and TM modes can be solved analytically which is not possible in

a mixed coordinate system. The general procedure to extract the S-parameters is based on the radial mode matching technique in propagation direction of the wave.

Although normally the ridges in a CRW are assumed to be rectangular (to simulate tuning screws), a conical ridge shape, such as that considered in this paper (Figure 1), does not deteriorate the electrical performance of the component nor does it complicate the fabrication, but the numerical modelling of the ridge section simplifies. In some applications it was even found that the shape of the ridges (for ridge thickness $< 200\mu\text{m}$), whether rectangular or conical, is irrelevant in practise [1].

To illustrate the efficiency of the numerical approach presented in this paper we have optimized a three section circular double ridged filter in a below-cutoff waveguide and also a three section waveguide transformer.

II Theory

A radial mode matching method is developed to calculate the generalized scattering matrix of a discontinuity between an empty circular waveguide and a double ridged circular waveguide. The cross section of such a discontinuity is shown in Figure 1. Since the TE_{11} mode is the fundamental mode of propagation a magnetic and electric wall symmetry can be used. The electric and magnetic vector potential functions in the empty circular waveguide for such a symmetry can be written as follows

$$\psi^{(h)} = \sum_{m=1}^M \sum_{n=1,3}^N P_{n,m} J_n(k_{c_n}^{Ih} \rho) \sin n\phi \quad (1)$$

$$\psi^{(e)} = \sum_{m=1}^M \sum_{n=1,3}^N Q_{n,m} J_n(k_{c_n}^{Ie} \rho) \cos n\phi \quad (2)$$

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The coefficients P and Q are the power normalization constants and are obtained by setting the magnitude of the power carried in each of the modes to unity. The values of M and

N depend on the number of TE and TM modes used in the evaluation of the generalized scattering matrix. The electric and magnetic vector potentials in the ridged circular waveguide subsections (1) and (2), for a ridge thickness of 2θ can be written as follows

$$\psi^{(1h)} = \sum_{r=1}^R \sum_{n=1,3}^{N1} A_n J_n(k_c^{IIh} \rho) \sin n\phi \quad (3)$$

$$\begin{aligned} \psi^{(2h)} = & \sum_{r=1}^R \sum_{m=0}^{N2} C_m [N'_l(k_c^{IIh} a) J_l(k_c^{IIh} \rho) - \\ & J'_l(k_c^{IIh} a) N_l(k_c^{IIh} \rho)] \cos l(\phi - \theta) \end{aligned} \quad (4)$$

where J denotes the Bessel functions and N the Naumann functions and $(')$ their derivatives; $l = \frac{m\pi}{2\theta}$

$$\psi^{(1e)} = \sum_{r=1}^R \sum_{n=1,3}^{N1} B_n J_n(k_c^{IIe} \rho) \cos n\phi \quad (5)$$

$$\begin{aligned} \psi^{(2e)} = & \sum_{r=1}^R \sum_{m=1}^{N2} D_m [N_l(k_c^{IIe} a) J_l(k_c^{IIe} \rho) - \\ & J_l(k_c^{IIe} a) N_l(k_c^{IIe} \rho)] \sin l(\phi - \theta) \end{aligned} \quad (6)$$

The procedure to determine the eigenvalues and the amplitude coefficients in the above equations has been described in [5,6]. The amplitude coefficients are once again power normalized so that the magnitude of the power carried in each of the modes is unity. For thin ridges, $N1$ is chosen to be equal to $N2$ and these values along with R depend on the number of TE and TM modes necessary to achieve convergence of the S-parameters.

From the potential functions described above in the two regions of discontinuity the electric and magnetic fields in each of the regions of Figure 1 can be derived. At the interface of the two regions ($z = 0$), the continuity of the tangential components E and H -field of the incident and reflected waves is applied. Using the orthogonality property of the modes, the continuity condition results in four sets of equations relating the unknown wave amplitudes of the incident(F) and reflected(B) waves. For instance, the continuity of tangential components of E -field results in the two sets of equations given below:

$$\begin{aligned} (F^{Ih} + B^{Ih}) &= [L_{HH}](F^{IIh} + B^{IIh}) \\ (F^{Ie} + B^{Ie}) &= [L_{EE}](F^{IIe} + B^{IIe}) + \\ & [L_{EH}](F^{IIh} + B^{IIh}) \end{aligned}$$

where $[L]$ matrices give the coupling between the fundamental and higher order TE and TM modes. The matrix equations can be rearranged suitably [4] and inverted to yield the

generalized scattering matrix of the discontinuity between empty circular waveguide and a ridged circular waveguide.

Structures like evanescent-mode filters also involve a step discontinuity between two axially symmetric circular waveguides. This procedure has been explained elsewhere in literature and will not be repeated here. By cascading generalized scattering matrices [2]-[4] at various step discontinuities the generalized S-matrix of matching networks and evanescent-mode filters are obtained.

III Results

Before designing filters and impedance transformers, the algorithm developed in this paper is tested first with respect to convergence and measured results for individual discontinuities. Since an iris step from a large to a small circular waveguide is part of the evanescent filter of Figure 5, this transition has been tested first and the results are given in Table 1. Good agreement was found with measurements given in [8]. To obtain this good agreement 40 TE and TM modes are necessary in the larger waveguide while 20 are necessary in the smaller. For a transition between a circular waveguide and a circular double ridged waveguide up to 40 TE and TM modes are necessary on both sides of the discontinuity (Figure 2). Since for this discontinuity no measurement results are available in the open literature, we have fabricated the two ridges by cutting a thin brass plate (thickness $125\mu\text{m}$) and inserted them into a split-block circular waveguide housing. As illustrated in Figure 3, both measured and calculated results are in good agreement. The slight deviations are probably due to the difficulty in positioning both ridges symmetrically into the split-block housing and approximation in the calculation by a conically shaped ridge with angular thickness of $\theta = 1$ degree.

On the basis of the above investigation, we have then designed a three section Chebyshev transformer in a double ridged circular waveguide. The initial design was based on the characteristic impedance of the fundamental mode. The optimization with respect to the lengths of the transformer sections was performed with 20 modes and a final analysis was done with 40 TE and TM modes. The results are shown in Figure 4. As a second example we have designed an evanescent-mode filter (Figure 5). The lengths of the ridge and evanescent sections are altered until an optimum filter response is obtained. During the optimization 30 modes are considered while in the final analysis 40 modes are included.

The optimization routine utilized in this work is based on a very reliable practical Quasi-Newton algorithm described in [7]. This algorithm uses a slightly modified version of Fletcher's inexact line search.

IV Conclusions

This paper has introduced a radial mode matching technique for field theory design of circular ridged waveguide components like filters and impedance transformers. Fundamental and higher order mode interaction at and between discontinuities has been taken into account. Good agreement between theoretical and measured results have been found.

References

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Table 1
Calculated and measured S parameters
(radius of input/output section=0.50175
radius of the iris=0.25 in; f=9.0 GHz)

Thickness (inch)	Reflection Coefficient s_{11}			
	Calculated		Measured	
	Magnitude	Phase (o)	Magnitude	Phase (o)
0.005	0.837	150.5	0.855	150.5
0.008	0.881	151.1	0.866	151.7
0.050	0.938	156.5	0.927	155.3
0.100	0.968	159.3	0.956	158.1
0.200	0.990	161.6	0.981	160.6
0.500	0.999	162.6	0.993	161.1
1.000	1.000	162.6	0.995	161.5
Transmission Coefficient s_{12}				
0.005	0.488	60.5	0.465	56.8
0.008	0.474	61.1	0.451	59.3
0.050	0.345	66.4	0.330	62.6
0.100	0.250	69.3	0.240	65.1
0.200	0.138	71.6	0.134	67.1
0.500	0.025	72.6	0.026	69.0
1.000	0.002	72.6	0.002	70.1

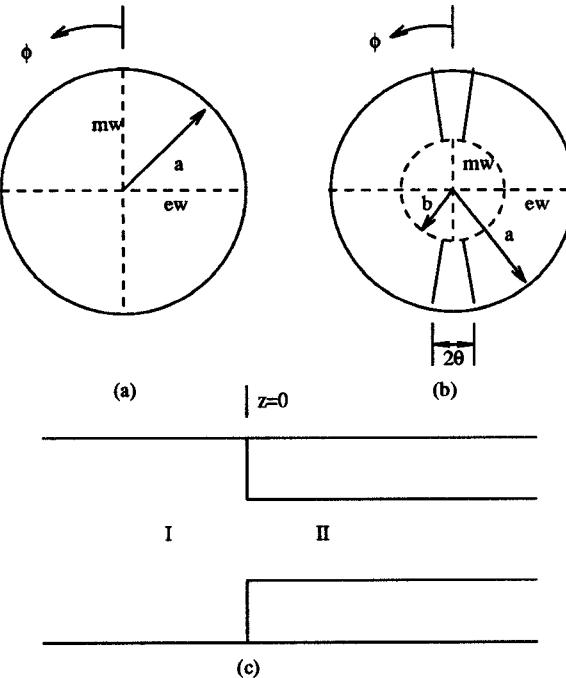


Figure 1: Discontinuity regions (a) Circular waveguide (region I) (b) Ridged circular waveguide (region II) (c) Longitudinal cross-section

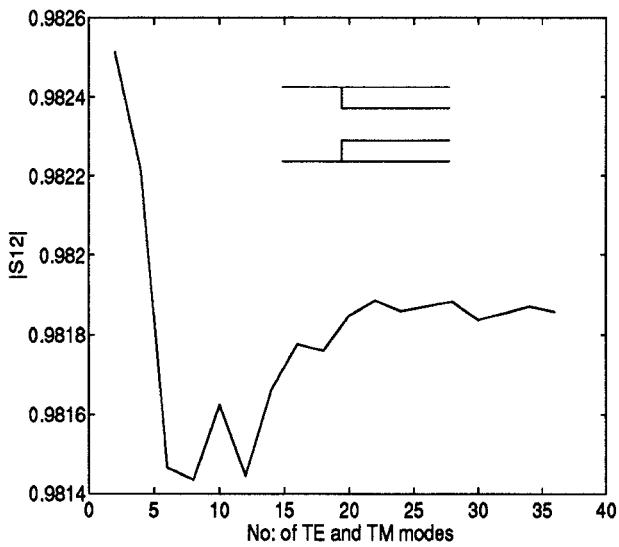


Figure 2: Magnitude of S_{12} of a discontinuity from a circular waveguide to ridged circular waveguide, $a=2\text{mm}$, $b=1.5\text{mm}$, $f=55\text{GHz}$

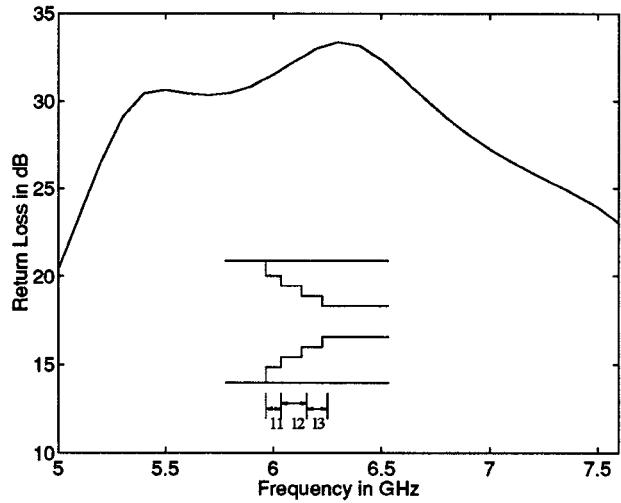


Figure 4: Response of an optimum 3-section ridged circular waveguide transformer, $\theta = 5\text{degree}$, dimensions in cm, section 1: $a=2$, $b=1.7$, $l1=1.633$, section 2: $a=2$, $b=1.13$, $l2=1.351$, section 3: $a=2$, $b=0.7$, $l3=1.191$, section 4: $a=2$, $b=0.5$

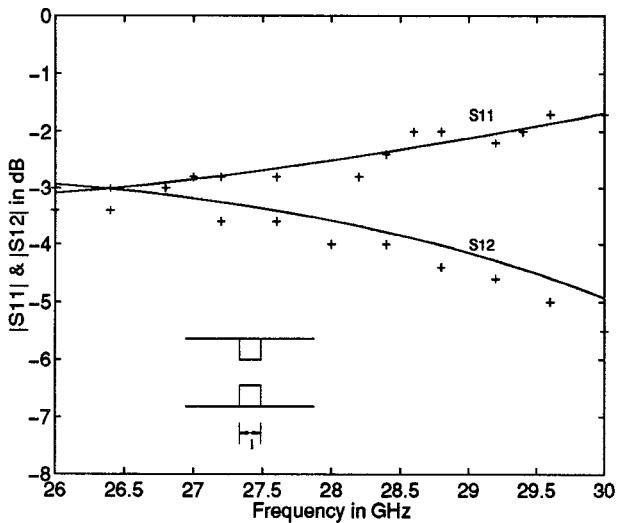


Figure 3: S-parameters in dB of a discontinuity from circular waveguide to ridge circular waveguide of finite length, $a=4\text{mm}$, $b=2\text{mm}$, ridge thickness = $125\mu\text{m}$ ($\theta = 1\text{ degree}$), $l=1.1\text{mm}$, + measured, - calculated

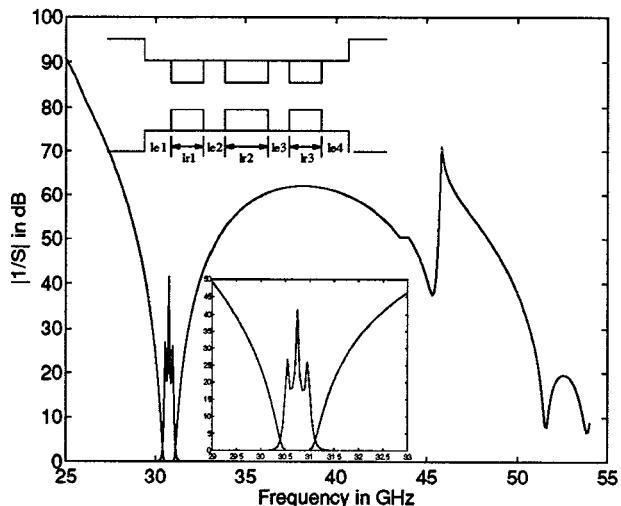


Figure 5: Response of an optimum 3-resonator evanescent-mode circular waveguide filter, dimensions in mm, radius of input/output section=4, radius of evanescent section(a)=2, radius of resonator section(b)=0.4, $\theta = 5\text{deg}$, $le1=le4=1.152$, $lr1=lr3=1.679$, $le2=le3=4.423$, $lr2=1.911$